解析学III　　確認課題 No.301　正解　　平成　年度

[出題項目 復習]

問１．次ぎの関数を微分せよ。

(1) $y = e^{-x}\sin x$ **(2)** $y = a^{2x^2}$ **(3)** $y = \log_e|\cos x|$ **(4)** $y = \arcsin\sqrt{x}$ **(5)** $y = \arctan\dfrac{2x+3}{4}$

[正解] (2) $t = 2x^2$ とおくと，$\dfrac{dy}{dx} = \dfrac{d}{dt}(a^t)\times\dfrac{dt}{dx} = a^t\log_e a\times 4x = 4xa^{2x^2}\log_e a$

(3) $t = \cos x$ とおくと，$\dfrac{dy}{dx} = \dfrac{1}{t}\times\dfrac{dt}{dx} = -\dfrac{\sin x}{\cos x} = -\tan x$

(4) $t = \sqrt{x}$ とおくと，$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-t^2}}\times\dfrac{dt}{dx} = \dfrac{1}{\sqrt{1-x}}\times\dfrac{1}{2\sqrt{x}} = \dfrac{1}{2\sqrt{x(1-x)}}$

(5) $t = \dfrac{2x+3}{4}$ とおくと，$\dfrac{dy}{dx} = \dfrac{1}{1+t^2}\times\dfrac{dt}{dx} = \dfrac{1}{1+(\frac{2x+3}{4})^2}\times\dfrac{1}{2} = \dfrac{8}{4x^2+12x+25}$

問２．次の定積分を (原始関数を用いて) 求めよ。

(1) $\displaystyle\int_{-1}^{0}\cos\frac{\pi}{3}x\ dx$ **(2)** $\displaystyle\int_{1}^{e}\frac{\log_e x}{x}\ dx$ **(3)** $\displaystyle\int_{0}^{\frac{\pi}{2}}\cos^2 x\ dx$

(4) $\displaystyle\int_{0}^{\pi}x\sin x\ dx$ **(5)** $\displaystyle\int_{1}^{2}x\log_e x\ dx$ **(6)** $\displaystyle\int_{0}^{2}xe^{\frac{x}{2}}\ dx$

(7) $\displaystyle\int_{0}^{\frac{\pi}{2}}-\sin x\cos^3 x - 3\sin x\cos x\ dx$ **(8)** $\displaystyle\int_{0}^{\frac{\pi}{3}}\frac{\sin^3 x}{1+2\cos x}\ dx$ **(9)** $\displaystyle\int_{0}^{\frac{\pi}{4}}\frac{1}{\cos^4 x}\ dx$

[正解] (1) $\displaystyle\int_{-1}^{0}\cos\frac{\pi}{3}x\ dx = \frac{3}{\pi}[\sin\frac{\pi}{3}x]_{-1}^{0} = \frac{3}{\pi}(\sin 0 - \sin\frac{-\pi}{3}) = \frac{3}{\pi}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}$

(2) $t = \log_e x$ とおくと，$dt = \frac{1}{x}dx$ で $x = 1$ のとき $t = 0$ かつ $x = e$ のとき $t = 1$ となる。

これらより，$\displaystyle\int_{1}^{e}\frac{\log_e x}{x}\ dx = \int_{1}^{e}\log_e x\times\frac{1}{x}\ dx = \int_{0}^{1}t\ dt = \frac{1}{2}[t^2]_0^1 = \frac{1}{2}$

(3) $\displaystyle\int_{0}^{\frac{\pi}{2}}\cos^2 x\ dx = \int_{0}^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}\ dx = \frac{1}{4}[2x+\sin 2x]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

(4) $\displaystyle\int_{0}^{\pi}x\sin x\ dx = [-x\cos x]_0^{\pi} + \int_{0}^{\pi}\cos x\ dx = -\pi\cos\pi + [\sin x]_0^{\pi} = \pi$

(5) $\displaystyle\int_{1}^{2}x\log_e x\ dx = \frac{1}{2}[x^2\log_e x]_1^2 - \frac{1}{2}\int_{1}^{2}x\ dx = \frac{1}{2}\times 4\log_e 2 - \frac{1}{4}[x^2]_1^2 = 2\log_e 2 - \frac{3}{4}$

(6) $\displaystyle\int_{0}^{2}xe^{\frac{x}{2}}\ dx = 2[xe^{\frac{x}{2}}]_0^2 - 2\int_{0}^{2}e^{\frac{x}{2}}\ dx = 4e - 4[e^{\frac{x}{2}}]_0^2 = 4e - 4e + 4 = 4$

(7) $\displaystyle\int_{0}^{\frac{\pi}{2}}-\sin x\cos^3 x - 3\sin x\cos x\ dx = \int_{0}^{\frac{\pi}{2}}-\sin x\cos x(\cos^2 x+3)\ dx$

ここで，$t = \cos x$ とおくと，$dt = -\sin x dx$，$x = 0$ のとき $t = 1$ かつ $x = \frac{\pi}{2}$ のとき $t = 0$ となる。これらより，$\displaystyle\int_{0}^{\frac{\pi}{2}}-\sin x\cos x(\cos^2 x+3)\ dx = \int_{1}^{0}t(t^2+3)\ dt = [\frac{1}{4}t^4+\frac{3}{2}t^2]_1^0 = -\frac{1}{4}-\frac{3}{2} = -\frac{7}{4}$

(8) $t = \cos x$ とおくと，$dt = -\sin x dx$ で $x = 0$ のとき $t = 1$ かつ $x = \frac{\pi}{3}$ のとき $t = \frac{1}{2}$ となる。これらより，$\displaystyle\int_{0}^{\frac{\pi}{3}}\frac{\sin^3 x}{1+2\cos x}\ dx = \int_{0}^{\frac{\pi}{3}}\frac{1-\cos^2 x}{1+2\cos x}\times\sin x\ dx = \int_{1}^{\frac{1}{2}}\frac{t^2-1}{2t+1}\ dt$

$\displaystyle= \int_{1}^{\frac{1}{2}}(\frac{1}{2}t - \frac{1}{4} - \frac{3}{4(2t+1)})\ dt = [\frac{1}{4}t^2 - \frac{1}{4}t - \frac{3}{8}\log_e|4(2t+1)|]_1^{\frac{1}{2}} = -\frac{1}{16} + \frac{3}{8}\log_e\frac{3}{2}$

(9) $t = \tan x$ とおくと，$dt = \dfrac{1}{\cos^2 x}dx$ で $x = 0$ のとき $t = 0$ かつ $x = \frac{\pi}{4}$ のとき $t = 1$ となる。これらより，$\displaystyle\int_{0}^{\frac{\pi}{4}}\frac{1}{\cos^4 x}\ dx = \int_{0}^{\frac{\pi}{4}}\frac{1}{\cos^2 x}\times\frac{1}{\cos^2 x}\ dx = \int_{0}^{\frac{\pi}{4}}(1+\tan^2 x)\times\frac{1}{\cos^2 x}\ dx$

$\displaystyle= \int_{0}^{1}(1+t^2)\ dt = [t+\frac{1}{3}t^3]_0^1 = 1+\frac{1}{3} = \frac{4}{3}$